Optimal prioritized channel allocation in cellular mobile systems

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Abstract

Under the cutoff priority discipline, the optimal prioritized channel allocation problem is formulated, which minimizes the weighted average blocking probability of handoff calls while ensuring the prespecified grade of service for new calls and the co-channel interference constraints. We use the concept of pattern to deal with the problem more conveniently. Using Lagrangean relaxation and subgradient optimization techniques, we obtain high-quality solutions with information about their deviations from true optimal solutions. Computational experiments show that our method works very well.

Scope and purpose

Channel allocation is one of the most important problems in the design of cellular mobile systems. Since the number of cells of forthcoming networks will increase rapidly, this problem will be of even greater importance in the future. In cellular mobile systems, a new channel should be assigned by the new base when a call enters an adjacent cell. It provides continuation of ongoing calls as the user travels across cell boundaries, and is called handoff. The handoff call is forced to terminate before completion if there are no channels available in the new cell. However, in many practical situations, the blocking of a handoff call attempt is critical since it will result in a disconnection of the call in the middle of conversation. Thus, for reducing the blocking probability of handoff calls, several algorithms based on the cutoff priority scheme have been introduced in the literature. In the cutoff priority scheme, priority is given to handoff calls by exclusively reserving some channels called guard channels for them. In this article, we formulate a prioritized channel allocation problem under the cutoff priority scheme in general multicell environments, and suggest an efficient algorithm to solve that problem. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Prioritized channel allocation; Lagrangean relaxation; Subgradient method; Cellular mobile system

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1. Introduction

In cellular mobile systems, the service area is divided into a certain number of cells with various radii from a few hundred meters to tens of kilometers. A user communicates via a radio channel to the base station in a cell. Channel frequencies are reused in cells that are adequately separated in distance so that mutual interference is below tolerable levels, and no adjacent cells are assigned a common frequency. Thus, when a mobile user enters an adjacent cell, a new channel should be assigned by the new base. It provides continuation of ongoing calls as the user travels across cell boundaries, and is called handoff. The handoff call is forced to terminate before completion if there are no channels available in the new cell.

However, in many practical situations, the blocking of a handoff call attempt is critical since it will result in a disconnection of the call in the middle of conversation. Thus, for reducing the blocking probability of handoff calls, several algorithms based on the cutoff priority scheme (CPS) have been introduced in the literature [1–5]. In CPS, priority is given to handoff calls by exclusively reserving some channels for them. The reserved $y_i$ channels among $C_i$ channels available in cell $i$ are referred to as the guard channels, and the remaining $C_i - y_i$ (=$x_i$) channels, called the ordinary channels, are shared by both calls. When a new call attempt is generated in cell $i$, it is blocked and cleared if the number of free channels is less than or equal to $y_i$. But, a handoff call attempt fails only when all the $C_i$ channels are busy in the cell. CPS is not commonly being utilized in operational systems today. However, since it is very simple to implement and its impact on the system performance is straightforward, any cellular operator can consider introducing it in presently operational systems if reducing handoff failure becomes a more important design issue.

The penalty of implementing CPS is the increment in the blocking probability of new calls due to the fact that fewer channels are granted to them. This disadvantage should be assessed when CPS is considered for practical usage. To evaluate the blocking probability performance of CPS for multicell systems, we also need to solve a more complex channel allocation problem than the ordinary one [6,7, other ordinary channel allocation references]. Given a certain number of frequency channels and traffic loads, the channel allocation for CPS is to determine the numbers of guard channels as well as ordinary channels for each cell, which is referred to as the prioritized channel allocation in the following. In [8], prioritized channel allocation methods are presented only for a single-cell system and one cluster of a multi-cell system. In this paper, we consider the prioritized channel allocation problem in general multi-cell environments, which is to minimize the weighted average blocking probability of handoff calls while satisfying the prespecified grade of service (GOS) for new calls and the co-channel interference constraints.

To deal with the complex prioritized channel allocation problem conveniently, we reduce the problem using the concept of frequency reuse pattern [6,7,9]. The reuse pattern is a set of cells to which a channel can be allocated without causing co-channel interference. In particular, a pattern in which mutual distances between adjacent co-channel cells are equal to the minimum reuse distance is called minimum reuse pattern. This concept helps to avoid obtaining many equal symmetric solutions which may occur by permuting channels with unaltered value on the objective function. A Lagrangean relaxation procedure is constructed for solving the reduced prioritized channel allocation problem since this method may provide high-quality solutions with information about their error ranges. We have applied this scheme successfully and computational experiments show that our scheme finds high-quality solutions.
The remainder of the paper is organized as follows. In Section 2, the optimal prioritized channel allocation problem is defined and formulated mathematically, and a reduced problem using the concept of pattern is introduced. In Section 3, the solution procedure is described. In Section 4, the results of computational experiments are reported and some discussion on the performance of CPS is also made. Finally, Section 5 summarizes the results.

2. Prioritized channel allocation problem

We consider a cellular mobile system consisting of \( N \) cells and \( M \) channels. Let \( \lambda_i^n \) and \( \lambda_i^h \) be the traffic demands in erlangs of new and handoff calls in cell \( i \), respectively, and let \( x_i \) and \( y_i \) be the numbers of ordinary and guard channels available in cell \( i \), respectively. Then the blocking probabilities of new and handoff calls in the cell are, respectively, given by

\[
BN_i(\lambda_i^n, \lambda_i^h, x_i, y_i) = \frac{\lambda_i^n}{N+1} + \frac{\lambda_i^n \sum_{j=1}^{M} (\lambda_i^h)^j/(x_i + j)!}{\sum_{j=0}^{x_i} \lambda_i^n/j! + \lambda_i^n \sum_{j=1}^{y_i} (\lambda_i^h)^j/(x_i + j)!},
\]

and

\[
BH_i(\lambda_i^n, \lambda_i^h, x_i, y_i) = \frac{\lambda_i^n \sum_{j=0}^{x_i} (\lambda_i^h)^j/(x_i + j)!}{\lambda_i^n + \lambda_i^n \sum_{j=1}^{y_i} (\lambda_i^h)^j/(x_i + j)!},
\]

where \( \lambda_i = \lambda_i^n + \lambda_i^h \) [2,8]. And the weighted average blocking probability of handoff calls in the cellular mobile system is given by

\[
\sum_{i=1}^{N} w_i BH_i(\lambda_i^n, \lambda_i^h, x_i, y_i),
\]

where \( w_i = \lambda_i^h / \sum_{i=1}^{N} \lambda_i^h \) is the traffic weighting factor.

The prioritized channel allocation problem PCAP, which minimizes the weighted average blocking probability of handoff calls while ensuring sufficient level of GOS for new calls and the co-channel interference constraints, is as follows:

\[
PCAP \quad \min \sum_{i=1}^{N} w_i BH_i(\lambda_i^n, \lambda_i^h, x_i, y_i)
\]

s.t. \( BN_i(\lambda_i^n, \lambda_i^h, x_i, y_i) \leq B_{\text{max}} \) for \( i = 1, \ldots, N \),

\( x_i + y_i = \sum_{j=1}^{M} f_{ij} \) for \( i = 1, \ldots, N \),

\( f_{sj} + f_{ij} \leq 1 \) for \( j = 1, \ldots, M \), and all interfering cell pairs \((s,t)\),

\( x_i, y_i, f_{ij} \): nonnegative integer, for \( i = 1, \ldots, N \),

\( f_{ij} = 0 \) or \( 1 \) for \( i = 1, \ldots, N \), \( j = 1, \ldots, M \),

where \( B_{\text{max}} \) represents the prescribed level of GOS of new calls.
This is a nonlinear integer programming problem. The decision variable $f_{ij}$ is a binary integer variable indicating channel allocation where $f_{ij} = 1$ represents that channel $j$ is allocated to cell $i$ and $f_{ij} = 0$ otherwise. The decision variables $x_i$ and $y_i$, representing the number of ordinary and guard channels allocated to cell $i$, respectively, are determined by channel allocation $f_{ij}$’s as shown in Eq. (2). Eq. (3) means that the same channel $j$ should not be allocated to different cells $s$ and $t$ simultaneously if the cells $s$ and $t$ are within interference zone of each other.

To deal with the proposed problem more conveniently, we introduce the concept of pattern. A channel cannot be allocated to adjacent cells simultaneously because of the co-channel interference. If an identical channel is allocated to a set of cells without causing co-channel interference between pairs of cells, these cells are called co-channel cells. This set of co-channel cells forms a pattern.

Now suppose that we generate $P$ patterns such that every cell belongs to at least one of these patterns. Then our problem reduces to the problem of allocating $M$ channels to $P$ patterns. Let the decision variable $t_p$ denote the number of channels allocated to pattern $p$. Then using the $P$ patterns, the problem PCAP reduces to the following problem:

\[
\text{RPCAP} \quad \min \sum_{i=1}^{N} w_i BH_i(i^m, j^h, x_i, y_i) \\
\text{s.t.} \quad BN_i(i^m, j^h, x_i, y_i) \leq B_{max} \quad \text{for } i = 1, \ldots, N, \quad (6) \\
\sum_{p=1}^{P} t_p \leq M, \quad (7) \\
(x_i + y_i) \leq \sum_{p \in S_i} t_p \quad \text{for } i = 1, \ldots, N, \quad (8) \\
x_i, y_i: \text{ nonnegative integer, for } i = 1, \ldots, N, \quad (9) \\
t_p: \text{ nonnegative integer, for } p = 1, \ldots, P, \quad (10)
\]

where $S_i$ is the set of patterns which covers cell $i$. Here, the right-hand side of (8) is the number of channels available in cell $i$. If all feasible patterns are considered, the problem RPCAP is equivalent to the original problem PCAP. Due to the property of a pattern, a solution of the problem RPCAP always satisfies the co-channel interference constraints. This problem RPCAP avoids obtaining many equal symmetric solutions which may occur by permuting channels with unaltered value on the objective function in the problem PCAP.

When only a subset of the feasible patterns is considered, RPCAP is an approximation to the original problem PCAP. The choice of patterns considered in RPCAP can influence the quality of solutions relative to the original problem PCAP. Thus it is very important to find good candidate patterns. In [7], we have suggested three pattern generation procedures considering two facts: mutual distances between adjacent co-channel cells and the traffic demand distribution. These procedures find patterns such that mutual distances between adjacent co-channel cells are made short as far as possible, and patterns including cells with high traffic demands as much as possible.
3. Solution procedure

3.1. Lagrangean relaxation

We apply the Lagrangean relaxation method to RPCAP by relaxing constraints (8). Let \( \pi_i, i = 1, \ldots, N \), be the Lagrangean multipliers associated with constraints (8). Then the Lagrangean relaxation of problem RPCAP is

\[
\text{LRPCAP} \quad L(\pi) = \min \sum_{i=1}^{N} \left( w_i BH_i(\lambda^p_i, \lambda^h_i, x_i, y_i) + \pi_i \left( x_i + y_i - \sum_{p \in S_i} t_p \right) \right)
\]

\[
s.t. \quad x_i + y_i \leq M \quad \text{for} \quad i = 1, \ldots, N,
\]

Eqs. (6), (7), (9) and (10).

Note that Eqs. (7) and (8) automatically imply Eq. (11).

It is well known from optimization theory [10] that for each \( \pi \geq 0 \), \( L(\pi) \) is a lower bound of the optimal value of RPCAP. We are interested in obtaining the tightest possible lower bound, i.e., in the multipliers vector \( \pi^* \), which satisfies \( L(\pi^*) = \max_{\pi \geq 0} L(\pi) \).

3.2. Solving the relaxed problem LRPCAP

Since we placed the only constraints that link \( t \) with \( x \) and \( y \) in the objective, the problem LRPCAP, for the fixed \( \pi \), naturally breaks into two separate problems — one in \( x \) and \( y \) variables and one in the \( t \) variables.

Subproblem LRPCAP\((x,y)\): \[
\min \sum_{i=1}^{N} \left( w_i BH_i(\lambda^p_i, \lambda^h_i, x_i, y_i) + \pi_i (x_i + y_i) \right)
\]

\[
s.t. \quad BN_i(\lambda^p_i, \lambda^h_i, x_i, y_i) \leq B_{\max} \quad \text{for} \quad i = 1, \ldots, N,
\]

\[
x_i + y_i \leq M \quad \text{for} \quad i = 1, \ldots, N,
\]

\[
x_i, y_i: \quad \text{nonnegative integer, for} \quad i = 1, \ldots, N.
\]

This subproblem is decomposed into \( N \) problems over each cell \( i \), which can be easily solved by the following procedure.

**Step 1**: Find the smallest integer \( x_i^0 \) such that \( BN_i(\lambda^p_i, \lambda^h_i, x_i^0, 0) \leq B_{\max} \). Set \( k = x_i^0 \) and \( V_i^* = L \) where \( L \) is a relatively large value.

**Step 2**: Find the largest nonnegative integer \( l (\leq M - k) \) such that \( w_i (BH_i(\lambda^p_i, \lambda^h_i, k, l - 1) - BH_i(\lambda^p_i, \lambda^h_i, k, l)) > \pi_i \) and \( BN_i(\lambda^p_i, \lambda^h_i, k, y_i^0) \leq B_{\max} \), where \( BH_i(\lambda^p_i, \lambda^h_i, k, -1) = \infty \). Compute \( V_i = w_i BH_i(\lambda^p_i, \lambda^h_i, k, l) + \pi_i (k + l) \). If \( V_i < V_i^* \), then set \( V_i^* = V_i \), \( \bar{x}_i = k \) and \( \bar{y}_i = l \).

**Step 3**: If \( k = M \), then terminate with an optimal solution \( (\bar{x}_i, \bar{y}_i) \), otherwise, go to Step 2 replacing \( k \) by \( k + 1 \).

Note that we consider only the case of \( x_i \geq x_i^0 \) since \( BN_i(\lambda^p_i, \lambda^h_i, x_i, y_i) < BN_i(\lambda^p_i, \lambda^h_i, x_i, y_i + 1) \) [8], and in Step 2 we neglect the case of \( y_i > l \) since \( (BH_i(\lambda^p_i, \lambda^h_i, x_i, l - 1) - BH_i(\lambda^p_i, \lambda^h_i, x_i, l + 1)) \leq (BH_i(\lambda^p_i, \lambda^h_i, x_i, l - 1) - BH_i(\lambda^p_i, \lambda^h_i, x_i, l)) [8].
Subproblem LRPCAP(t): \[
\text{min} - \sum_{i=1}^{N} \pi_i \sum_{p \in S_i} t_p \\
\text{s.t.} \sum_{p=1}^{P} t_p \leq M,
\]
\[t_p: \text{ nonnegative integer, for } p = 1, \ldots, P.\]

Note that
\[
\sum_{i=1}^{N} \pi_i \sum_{p \in S_i} t_p = \sum_{i=1}^{P} \left( \sum_{p \in S_i} \pi_i \right) t_p = \sum_{p=1}^{P} \pi'_p t_p,
\]
where \(S'_p\) is the set of cells covered by the pattern \(p\) and \(\pi'_p = \sum_{i \in S'_p} \pi_i\). This subproblem is readily solved by setting to \(M\) a \(t_p\) variable that has the largest coefficient in the objective function and the remaining variables to 0, i.e.,
\[
\pi'_q = \max_p \{\pi'_p\} \Rightarrow t_p = \begin{cases} M & \text{if } p = q, \\ 0 & \text{otherwise}. \end{cases}
\]

3.3. Updating Lagrangean multipliers

We use the subgradient optimization method to solve the Lagrangean dual, i.e, to compute a vector of optimal Lagrange multipliers. Let \((\bar{x}, \bar{y}, \bar{t})\) be an optimal solution to the Lagrangean problem for a fixed vector \(\pi^k\). Then a subgradient \(\gamma^k = (\gamma^k_1, \gamma^k_2, \ldots, \gamma^k_N)\) for \(\pi^k\) is
\[
\gamma^k_i = (\bar{x}_i + \bar{y}_i) - \sum_{p \in S_i} \bar{t}_p \quad \text{for } i = 1, \ldots, N.
\]

We then update the Lagrange multipliers as
\[
\pi^{k+1} = \max(0, \pi^k + \sum_{j \in I_k} s^k_j \gamma^j),
\]
where \(I_k\) is a subset of indices taken from \(\{1, \ldots, k\}\) and \(s^k_j\) is a stepsize. Polyak [11] suggested a stepsize updating procedure and this procedure has been used for widespread practical applications of the algorithm [12]. Modifications of that procedure have been suggested to further improve computational efficiency [13,14]. Kim and Ahn [15] showed that a generalized subgradient algorithm for nondifferentiable convex programming converges to an optimal solution.

3.4. Finding a feasible solution

The solution of the relaxed problem is usually not a feasible solution of the original problem RPCAP. At each iteration, we try to obtain a good feasible solution from this relaxed solution by the following heuristic procedure.
Feasible solution finding procedure

Step 1: Let \((\tilde{x}, \tilde{y})\) be an optimal solution of the Lagrangean subproblem LRPCAP\((x, y)\) at the current iteration. Find the number of channels allocated to each pattern \(p\), \(t_p\), required to meet the channel demand of each cell \(i\), \(\tilde{x}_i + \tilde{y}_i\), i.e., obtain a heuristic feasible solution of the following integer problem:

\[
\text{IP } \min \sum_{p=1}^{P} t_p
\]

s.t. \(\sum_{p \in S_i} t_p \geq \tilde{x}_i + \tilde{y}_i \quad \text{for } i = 1, \ldots, N,\)

\(t_p:\) nonnegative integer, for \(p = 1, \ldots, P.\)

Step 2: For all \(i\), repeat Step 2.1 \((\sum_{p \in S_i} \hat{t}_p - (\tilde{x}_i + \tilde{y}_i))\) times, where \(\hat{t}_p\) is the solution obtained in Step 1.

Step 2.1: If \(BN_i(\lambda^n, \lambda^h, \tilde{x}_i, \tilde{y}_i + 1) \leq B_{\max}\), then set \(\tilde{y}_i = \tilde{y}_i + 1\), otherwise, \(\tilde{x}_i = \tilde{x}_i + 1\).

Step 3: Set \(d = M - \sum_{p=1}^{P} \hat{t}_p\). If \(d = 0\), then go to Step 4. If \(d > 0\), then repeat Step 3.1 \(d\) times, otherwise, repeat Step 3.2 \(|d|\) times.

Step 3.1: If \(BN_i(\lambda^n, \lambda^h, \tilde{x}_i, \tilde{y}_i + 1) \leq B_{\max}\), then set \(x_i = w_i(BH_i(\lambda^n, \lambda^h, \tilde{x}_i, \tilde{y}_i) - BH_i(\lambda^n, \lambda^h, \tilde{x}_i, \tilde{y}_i + 1))\) and \(I(i) = 0\), otherwise, set \(x_i = w_i(BH_i(\lambda^n, \lambda^h, \tilde{x}_i, \tilde{y}_i) - BH_i(\lambda^n, \lambda^h, \tilde{x}_i + 1, \tilde{y}_i))\) and \(I(i) = 1\). Compute \(\Delta_p\) for each pattern \(p\), where \(\Delta_p = \sum_{i \in S_i} x_i\). Set \(\Delta_q = \max_p \{\Delta_p\}\) and \(\tilde{t}_q = \tilde{t}_q + 1\). For \(i \in S_q\), if \(I(i) = 0\), then set \(\tilde{y}_i = \tilde{y}_i + 1\), otherwise, set \(\tilde{x}_i = \tilde{x}_i + 1\).

Step 3.2: If \(BN_i(\lambda^n, \lambda^h, \tilde{x}_i - 1, \tilde{y}_i) \leq B_{\max}\), then set \(x_i = w_i(BH_i(\lambda^n, \lambda^h, \tilde{x}_i - 1, \tilde{y}_i) - BH_i(\lambda^n, \lambda^h, \tilde{x}_i, \tilde{y}_i))\) and \(I(i) = 0\), otherwise, set \(x_i = w_i(BH_i(\lambda^n, \lambda^h, \tilde{x}_i, \tilde{y}_i - 1) - BH_i(\lambda^n, \lambda^h, \tilde{x}_i, \tilde{y}_i))\) and \(I(i) = 1\). Compute \(\Delta_p\) for each pattern \(p\), where \(\Delta_p = \sum_{i \in S_i} x_i\). Set \(\Delta_q = \min_p \{\Delta_p\}\) and \(\tilde{t}_q = \tilde{t}_q - 1\). For \(i \in S_q\), if \(I(i) = 0\), then set \(\tilde{x}_i = \tilde{x}_i - 1\), otherwise, set \(\tilde{y}_i = \tilde{y}_i - 1\).

Step 4: Terminate with a feasible solution \((\tilde{x}, \tilde{y}, \tilde{t})\) of the problem RPCAP.

In general, the solutions \((\tilde{x}, \tilde{y}, \tilde{t})\) of the Lagrangean subproblems do not satisfy the relaxed constraints \((8)\). To obtain a feasible solution using the solutions \((\tilde{x}, \tilde{y}, \tilde{t})\), Step 1 finds the number of channels allocated to each pattern \(p\), \(t_p\), required to meet the channel demand of each cell \(i\), \(\tilde{x}_i + \tilde{y}_i\). Step 2 increases \(\tilde{x}_i\) and \(\tilde{y}_i\) to satisfy the condition \(\tilde{x}_i + \tilde{y}_i = \sum_{p=1}^{P} \hat{t}_p\), where \(\hat{t}_p\) is the solution obtained in Step 1. This decreases the objective value of the problem RPCAP. If \(\sum_{p=1}^{P} \hat{t}_p\) is equal to the number of available channels, \(M\), then we have a good feasible solution \((\tilde{x}, \tilde{y}, \tilde{t})\), otherwise Step 3 increases or decreases \(\hat{t}_p\) and \((\tilde{x}_i, \tilde{y}_i)\) to obtain a feasible solution.

3.5. Summary of the overall solution procedure

Fig. 1 shows the flow diagram of the overall solution procedure in summary. First, the procedure initializes the Lagrangean multipliers \(x^0\). Then, for the fixed vector \(\pi\), the procedure solves the two subproblems LRPCAP\((x, y)\) and LRPCAP\((t)\), and updates the lower bound. After that, the procedure obtains a feasible solution using the Feasible Solution Finding Procedure, and updates
the incumbent solution. If the incumbent solution is within a pre-specified tolerance of the best lower bound or if the number of iterations has become excessive, the procedure terminates, otherwise the procedure updates the Lagrangean multipliers \( \pi^k \) and goes to the second step.

### 4. Computational experiments

We assess the numerical performance of the proposed scheme through comparisons with the pure CPS scheme and the ordinary scheme. These methods provide a benchmark of the performance of commonly deployed methods.

In the test, a \( 7 \times 7 \) regular hexagonal cellular mobile system is considered, and the minimum reuse distance \( d \) is assumed to be \( \sqrt{21}r \) where \( r \) is the cell radius, that is, the frequency reuse factor is 7 [16,17]. Cells having a mutual distance not smaller than \( d \) can use the same channels. Traffic demands for cells are randomly generated from Normal distribution, and the parameter \( B_{\text{max}} \) was set at 0.05. We consider the minimum reuse patterns in the computation, so that \( P = 14 \) [9]. In this case, the problem \( IP \) in Step 1 of the feasible solution finding procedure suggested in
Table 1
Computational results

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<th>No.</th>
<th>((m^<em>, v^</em>)^a)</th>
<th>((m^b, v^b)^b)</th>
<th>(M^c)</th>
<th>Lower bound(^d)</th>
<th>Upper bound(^e)</th>
<th>Error range ((%)^f)</th>
<th>Maximum b.p.(^g)</th>
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<td></td>
<td>0.008720</td>
<td>0.008960</td>
<td>2.75</td>
<td>0.038646</td>
</tr>
<tr>
<td>13</td>
<td>(4, 4)</td>
<td>(2, 2)</td>
<td>(R(96))</td>
<td>0.014151</td>
<td>0.014842</td>
<td>4.88</td>
<td>0.034796</td>
</tr>
<tr>
<td>14</td>
<td>(R + 5)</td>
<td></td>
<td></td>
<td>0.005444</td>
<td>0.005451</td>
<td>0.13</td>
<td>0.069494</td>
</tr>
</tbody>
</table>

\(^a\)\(m^*, v^*\): Mean and variance in erlangs of normal distribution for new calls.
\(^b\)\(m^b, v^b\): Mean and variance in erlangs of normal distribution for handoff calls.
\(^c\)\(M\): Number of channels considered.
\(^d\)Minimum number of channels necessary to satisfy GOS for new calls when only minimum reuse patterns are considered.
\(^e\)Objective function value of the best feasible solution.
\(^f\)(upper bound-lower bound) \times 100/lower bound.
\(^g\)The maximum amount of blocking for handoff calls in a cell.

Section 3 for obtaining a good feasible solution is easily solved since it is the dual of the classical assignment problem. Our algorithm terminates when \((f^c - L(\pi))/L(\pi)\) is less than a pre-specified tolerance \(10^{-7}\) where \(f^c\) is the current best-feasible objective value, or when the number of iterations exceeds the maximum number of iterations 500.

The computational results are summarized in Table 1 and Figs. 2–4. The problems in Table 1 and Figs. 3 and 4 are successfully solved in several minutes and in tens of minutes, respectively, on an IBM PC. Table 1 shows that our scheme obtains nearly optimal solutions for various traffic demands. For each case of traffic loads, we assume that \(R\) and \(R + 5\) channels are, respectively, available in the system, where \(R\) is the minimum number of channels required to satisfy GOS for new calls when only minimum reuse patterns are considered. The upper bound is the objective function value that is obtained by the solution. The error range to check a qualitative aspect of the solution is defined as (upper bound-lower bound) \times 100/lower bound. All the solutions are within 5% range of an optimal solution of the reduced problem RPCAP. The maximum blocking probability (b.p.) denotes the maximum amount of blocking for handoff calls in a cell. In all the cases except No. 1, our scheme provides better GOS for handoff calls than new calls while maintaining the GOS constraint for new calls. These results show that our scheme automatically guarantees GOS for handoff calls to the level of GOS for new calls in almost all the cases. However,
if a pre-specified level of GOS for handoff calls is definitely required, it is necessary to introduce the GOS constraint for handoff calls in problem PCAP. This problem can be easily solved using a procedure similar to the solution procedure proposed in Section 3.

It is noted that we numerically test the quality of solutions only considering the minimum reuse pattern. However, the minimum reuse pattern considered is equivalent to the minimum reuse distance constraint when the frequency reuse factor is 7, which is widely used in fixed channel planning practices. Thus, the results also support the quality of the solutions for the original problem where the minimum reuse distance constraints are employed.

Fig. 2 shows, for an illustrative purpose, the numbers of ordinary and guard channels assigned to each cell for the problem No. 11 in Table 1. The channels are not uniformly distributed according to randomly generated traffic loads. In some cells, no guard channels are assigned. This is because available channels are allocated as ordinary channels in order to maintain the required blocking probability of new calls.
Fig. 3 shows the performance of the proposed scheme compared with other channel allocation methods. Test problems are generated in more realistic environments where 336 channels are available and the largest traffic loads are offered as \((m^n, v^n) = (11.2, 11.2)\) and \((m^b, v^b) = (22.4, 22.4)\). Note that \((m^n, v^n)\) and \((m^b, v^b)\) represent pairs of mean and variance in erlangs of normal distribution for new calls and handoff calls, respectively. A cellular operator with 10 MHz frequency block has 333 channels if a channel requires 30 kHz bandwidth. However, the number of available channels is assumed to be 336 which is divided by the frequency reuse factor 7 for the convenience of tests. In the figure, \(CPS(LA)\) represents the proposed scheme based on the Lagrangean relaxation approach, and \(CPS(y_i)\) represents uniform channel allocation using \(CPS\) with \(y_i\) guard channels. Note that \(CPS(0)\) represents the ordinary uniform channel allocation scheme. The figure shows that our scheme obtains a good solution. Note that, in the case of \(CPS(0)\) and \(CPS(1)\), two cells and six cells do not satisfy the constraints (1), respectively. The proposed scheme achieves about 50–100% reduction in the blocking probability of handoff calls compared with the reference methods.

Fig. 4 shows the sensitivity to the GOS parameter \(B_{\text{max}}\). The traffic load is offered as \((m^n, v^n) = (11, 11)\) and \((m^b, v^b) = (22, 22)\), and the number of channels is 334, 336, and 338. The figure shows that the objective value decreases as the GOS parameter \(B_{\text{max}}\) increases.

5. Conclusions

The optimal prioritized channel allocation problem has been suggested, which minimizes the weighted average blocking probability of handoff calls while ensuring sufficient level of GOS for new calls and the co-channel interference constraints in a cellular mobile system with nonuniform traffic distribution. The problem has been converted into a simpler form through linearization and the concept of pattern. We have applied a Lagrangean relaxation procedure to the simplified problem to obtain lower bounds and feasible solutions. Computational experiments have shown that our scheme finds high-quality solutions.

In the tests, we have considered only the minimum reuse patterns. However, we have obtained a good solution in less computational effort since the minimum reuse patterns are good candidate...
patterns. In fact, if other patterns such as patterns generated by the pattern generation procedures in [7] are additionally considered, the solutions will be improved to some degree.

References


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