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# Non-stationary filtering methods for audio signals

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### ABSTRACT

This paper deals with filtering methods for audio signals using time frequency analysis. The concept of time frequency filtering is vital for the enhancement of non-stationary signals and systems. Time frequency filtering is performed as masking or convolution in the time frequency domain and is based on non-parametric modeling using direct convolution or multiplication. The application of filtering in the time frequency domain for the realisation of artificial reverberation is presented.

## INTRODUCTION

The generalisation of one dimensional filter theory to the time-frequency (TF) plane provides a powerful tool for the construction of non-stationary signals and for the simulation of complex time variable systems. In this paper we discuss the equivalence of one dimensional signal filtering operations with filtering operations in the TF plane using particular time frequency representations. Such operations include convolution and modulation in the TF domain. Two general types of TF filters are considered: first, TF masking filters and, second, TF convolution filters. This type of processing also includes synthesis of time histories from known distributions, which have been transformed by a filtering operation (masking or convolution) in the TF plane.

These methods can provide tools for the enhancement of non-stationary signals, such as audio signals. Considering the output of a time varying system, the interaction between the input signal and the time varying system can be regarded as an operation in the TF domain between the TF expansion of the signal and the TF response of the system.

The application of TF filtering in non-stationary signals is studied in the context of an example of artificial reverberation. Artificial reverberation is used for the enhancement of audio signals and is usually implemented as convolution between the input signal and the impulse response of a filter in the time domain. FIR and IIR filters have been used for this purpose [1-3]. In the literature models that describe the parametric representation of reverberation have been proposed [4]. Recently the design of reverberation based on psychoacoustics criteria has been studied [5]. In this work we propose the use of TF filtering as a method for the realisation of time varying frequency dependent reverberation.

## LINEAR TIME INVARIANT FILTERING

Conventional linear time invariant filtering can be expressed in the time domain form as the convolution integral

$$y(t) = \int_{-\infty}^{t} h(t-t_1)\,x(t_1)\,dt_1$$

where $x(t)$ is the input signal, $y(t)$ is the output signal and $h(t)$ is the impulse response of the filter. This one dimensional filtering operation is shown in Fig. 1.

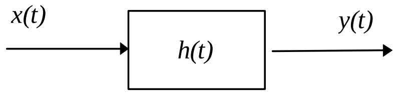


Figure 1. One dimensional convolution in the time domain.

The filtering action is more easily defined in relation to decomposing both *x(t)* and *h(t)* in the frequency domain where the filtering operation can be expressed as:

$$Y(\omega) = H(\omega)X(\omega)$$

where $H(\omega)$ is the frequency response of the time invariant system. From this expression we can more easily distinguish what parts of $X(\omega)$ can be removed or changed when compared to the time domain expression. This is one dimensional filtering realised as a product (or *masking*) operation in the frequency domain (Fig. 2).

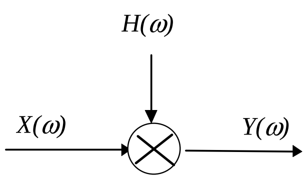


Figure 2. One dimensional filtering in the frequency domain.

These relations are useful for a wide range of applications but they have limitations when applied to non-stationary signals and systems.

## TIME-FREQUENCY FILTERING

The concept of time frequency filtering can be applied to a number of linear and quadratic distributions. The filtering relations of the Wigner distribution were demonstrated first in [6]. General TF filtering methods have been presented in [7,8] using optics, which could apply to a number of TF distributions. A method for the simulation of dynamic systems using linear distributions has been developed in [9]. The Karhunen-Loeve decomposition has been used for filtering by projecting the signal decomposition into a TF *subspace* in [10,11]. This method requires the solution of an eigenproblem and can be extended to reconstructing the signal using filter banks [12].

This idea is generalised to filtering operations in the TF plane and synthesis of signals from modified TF distributions, which yields classes of TF filters in two dimensions:

(i) *TF masking filters*, which are able to perform localised selection of TF components
(ii) *TF convolution filters*, which is a filtering method using convolution in the TF domain.

These two general classes provide a variety of possible operations in the TF plane.

However, there are some constraints on filtering of TF distributions, which are related to the *representability problem* of the distribution of the output. A modified distribution may not correspond to a realisable signal. There are several examples of filtered distributions, which do not correspond to a time history [8,13]. In general the constraints can be distinguished as distribution independent and distribution dependant.

A schematic presentation of the basic elements of filtering in the TF plane is shown in Fig. 3. The signals are expanded into the TF domain, then they are filtered by a one or two dimensional operation and the product is synthesised in the time domain using the corresponding synthesis formulae. The TF analysis $S_x(t,\omega)$ of a signal *x(t)* is transformed by the operator $M(t,\omega)$. This mask may process the signal in the same way as a time variable filter. In such cases we relate the mask to the non-stationary system using the two dimensional transfer function $H(t,\omega)$.

The masking or convolution operation yields the expansion $S_y(t,\omega)$ of the response *y(t)*. It is desirable to be able to reconstruct the time history after a filtering operation. The result of the operation in the TF plane should be a representable (*valid)* TF distribution for the resulting signal (i.e. there exists a signal which corresponds to the TF distribution). This condition is not satisfied for all distributions and operations. However we may synthesise a signal by approximating a realisable distribution in a mean square sense [14]. This has to be close to the distribution of the output. A second possibility is to synthesise a signal whether or not the distribution is representable. Synthesis approaches can be found in [15, 16].

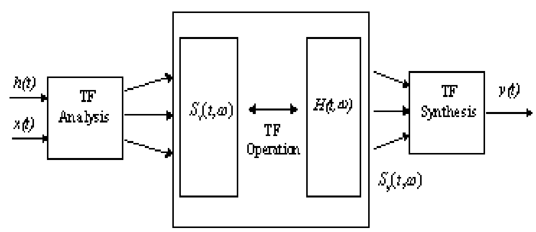


Figure 3. Schematic presentation of a general TF operation.

### Time-Frequency Masking Filters

Consider the construction of a filter within a given region on the TF domain. This is a filter that passes all signals located inside this region and rejects the ones located outside. The output is the product of a linear time varying filter, which is referred to as a *TF masking filter*. This can be visualised as the superposition of two TF surfaces with the system function to *pass or reject* signals according to a given region. Hence the output energy is zero for components outside the pass region.

$$S_y(t,\omega) = S_x(t,\omega)M(t,\omega)$$

The multiplication is analogous to the linear time invariant filter where the filter function $H(\omega)$ is multiplied with $X(\omega)$ in the frequency domain. The difference is that the instantaneous spectra of the signal and the mask are multiplied when using the TF masking operation. This can be visualised as a product operation applied along lines parallel to the frequency axis for every time instant *t*.

$$S_y(t_i,\omega) = S_x(t_i,\omega)M(t_i,\omega), \text{ for } -\infty<i<\infty$$

A schematic representation of this operation is shown in Fig. 4 considering a simplified TF distribution consisting of values 0 and 1. The expansion of a time variable signal x(t) and the mask are shown in Fig. 4.a and the result of masking in Fig. 4.b.

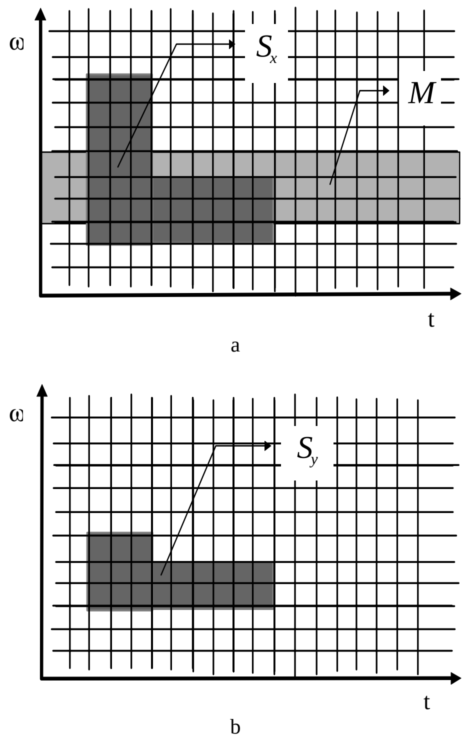


Figure 4. Masking in the TF plane.

This class of filters can be used to simulate time varying systems, which operate by selecting desired components in the TF domain or removing unwanted contributions such as noise [17,18]. A restriction applies which requires slow variation of the mask because of modulation effects. Additionally one has to be aware that this may be a rather crude way of filtering. This is because essential parts of the distribution, necessary for reconstruction, may be lost resulting in distortion.

### Time-Frequency Convolution Filters

*TF convolution filters* perform a transformation rather than a simple masking in the TF plane. This transformation can be a simple repositioning of the energy or even more complicated modifications. The filtering mask *M* has a different interpretation compared to the previously discussed case of masking filters and it can be expressed in the form:

$$S_y(t,\omega) = S_x(t,\omega) ** M(t,\omega)$$

where ($**$) denotes the set of operations in the TF domain consisting of the two dimensional convolution and the ordinary one dimensional convolution which can be performed along the time or the frequency axes. This class of filters performs transformation rather than selection, within the defined TF region or even between different regions in the TF domain.

Three types of TF convolution filters are defined for this set of operations according to how the *mask* $M(t,\omega)$ is convolved with a signal expansion in the TF domain.

(i) convolution along the time axis with respect to the time variable, corresponding to filtering in the time domain,
(ii) convolution along the frequency axis with respect to the frequency variable, corresponding to frequency modulation and
(iii) two dimensional convolution along the time and frequency axes, corresponding to mapping between certain classes of TF distributions.

### Convolution along the time axis

For a linear time invariant system convolution in the time domain is equivalent to convolution in the TF domain with respect to the time variable *t*. This is true if the TF distributions used for this operation preserve the property of convolution [7].

TF convolution along the time axis is defined as:

$$S_y(t,\omega) = \int M(t'-t,\omega) S_x(t',\omega) dt'$$

where the output in the time domain is given by the convolution $y(t) = h(t) * x(t)$ and its power spectral density is the product $S_y(\omega) = S_h(\omega) S_x(\omega)$.

The mask $M(t,\omega)$ acts as a filter on the time-frequency decomposition of $x(t)$ between corresponding bandlimited components of the decomposition (Fig. 5).

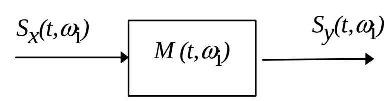


Figure 5. Convolution along the time axis in the TF plain.

This can be interpreted as filtering in time for each frequency $\omega_i$ using a superposition of bandlimited and time variable filters. It is desirable to model these components individually. If the TF distribution used for the operation satisfies the property of convolution along the time axis and the distributions involved in the operation are also valid, then, the product of the TF operation is representable (valid).

## IMPLEMENTATION OF TIME-FREQUENCY FILTERS

Linear TF distributions and specifically the Fourier transform is employed in order to simulate filtering operations in the TF plane. TF filtering requires the evaluation of the decompositions of the input signal, the system response and the inverse transform of the output signal and it is computationally expensive. To accelerate the computation we employ a recursive algorithm for the evaluation of the TF distributions, which is faster compared to non-recursive realisations and yields a simple synthesis procedure. In particular a recursive implementation of the Fourier transform is used.

### Recursive Implementation of the Fourier Transform

The method of computing the TF distribution of the input follows Papoulis [19] who proposed a recursive digital implementation of a rectangular window short time Fourier transform. The *running Fourier transform* of a signal has been defined as the integral

$$F_x(t,\omega) = \int_{-c}^{c} x(t+\tau) e^{-j\omega\tau} d\tau$$

with the inversion formula

$$x(t) = \frac{1}{2c}\sum_m F_x(t, m\omega_0), \qquad \omega_0 = \frac{\pi}{c}$$

where $c$ is a given constant describing the limits of the segment of analysis for a fixed $t$. $F(t,\omega)$ are the Fourier transformation coefficients, with respect to $\tau$, of the segment $x(t+\tau)$, $-c \le t \le c$ of $x(t)$. The above definition has been chosen because it leads to a recursive formulation. The evaluation of the spectrogram directly from this expression requires computation of the Fourier transform at any time instant $t$. It can be proved [20] that $F(t,m\omega_0)$ satisfies the first order differential equation.

$$\frac{dF_x(t,m\omega_0)}{dt} - jm\omega_0 F_x(t,m\omega_0) = (-1)^m x[(t+c) - x(t-c)]$$

In order to reduce the computation time, $F_x$ can be evaluated recursively.

Implementing in discrete time, the *running z-transform* is defined as the short time $z$-transform of a delayed signal. For a sequence $x(n)$, the running $z$-transform is:

$$\Phi(n,z) = \sum_{k=0}^{N-1} x(n-k)z^{-k}$$

For a fixed $n$, $\Phi(n,z)$ is the $z$-transform in the variable $k$ of the segment $x(n-k)$ of $x(n)$. The inversion formula, considering evaluation on the unit circle, is

$$x(n) = \frac{1}{N}\sum_{m=0}^{N-1}\Phi(n, w^{-m}) \quad \text{where} \quad w = e^{j\frac{2\pi}{N}}$$

Using this definition it is easy to recognise in the running $z$-transform the sampled version of the Fourier transform of a delayed sequence $x(n-k)$. For simplicity the above formulation assumes a rectangular window function applied to the signal:

$$g(n) = \begin{cases} 1 & for \quad 0 \le n \le N-1 \\ 0 & elsewhere \end{cases}$$

Using the running $z$-transform, with the substitution $k+1=p$, results in

$$\Phi(n-1, z) = z\sum_{p=1}^{N} x(n-p)z^{-p}$$

It follows that function $\Phi(n, w^{-m})$, which is the product of the running $z$-transform of $x(n)$ satisfies the first order recursion equation

$$\Phi(n,z) - z^{-1}\Phi(n-1,z) = x(n) - z^{-N}x(n-N)$$

and with $z = w^{-m}$, function $\Phi(n, w^{-m})$ has the simple recursive form:

$$\Phi(n,w^{-m}) - w^m\Phi(n-1,w^{-m}) = x(n) - x(n-N)$$

This defines a discrete recursive system with input $x(n)$, output $\Phi(n, w^{-m})$ and system function

$$S(m,z) = \frac{1 - z^{-N}}{1 - w^m z^{-1}}$$

The system consists of one shift register with output $x(n-N)$, one delay element and one multiplier (Fig. 6). Connecting $N$ such systems together in parallel, results in a running Discrete Fourier Series (DFS) spectrum analyzer which can be realised using filter bank structure (Fig. 7).

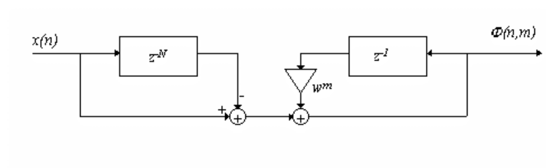


Figure 6. Elementary filter structure.

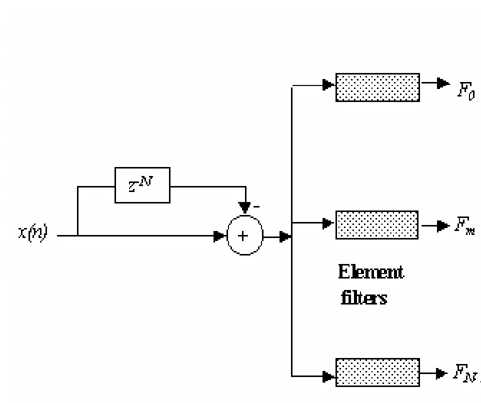


Figure 7. Recursive DFS analyser.

Prior to reconstructing the signal from its Fourier series expansion, it is possible to filter the TF coefficients according to a time variable system function implementing non-stationary TF convolution or masking filters.

Assuming that $H(n,z)$ is time-variant it is possible to modify the DFS coefficients of the input signal in order to obtain a dynamic time-frequency filter reconstructing a filtered version in the time domain.

$$y(n) = \frac{1}{N}\sum_{m=0}^{N-1}\Phi_x(n,w^{-m})H(n,w^{-m})$$

This corresponds to a TF masking operation between the *running* DFS coefficients of the input signal and the frequency samples of $H(n,z)$ (Fig. 8).

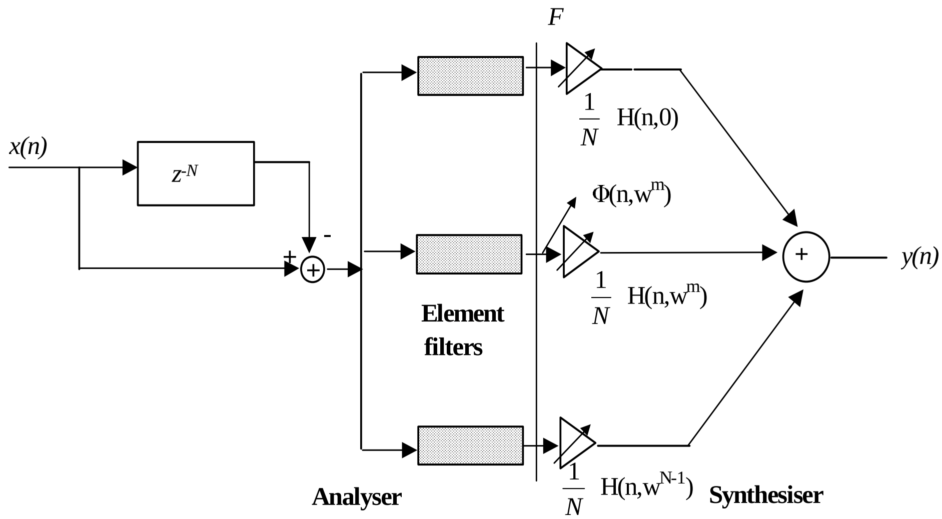


Figure 8. The filter bank for analysis, filtering and synthesis.

## EXAMPLE: REVERBERATION

The recursive implementation of the Fourier transform is used to add time variable non-uniform reverberation to a signal. The operation has been performed in the TF domain as convolution over the time axis. The TF convolution operation has been realised as a finite summation.

$$RDFS_y(n,m) = \sum_{n'}^{n-1} RDFS_h(n-n',m) RDFS_x(n',m)$$

The input signal and the impulse response of the system are decomposed and the corresponding sub-band signals are convolved for each band. The resulting decomposition provides the time history of the output using RDFS synthesis.

The mask has been designed using a number of impulsive signals. This has been generated in such a manner that corresponds to a system, which adds one reflection of lower amplitude than the original but the time when the reflections occur and their amplitude are functions of frequency. The RDFS decomposition of the input impulse seen in Fig. 9 has been TF filtered by the mask seen in Fig. 10. This has been realised by using TF convolution along the time axis. The result of the operation is seen to be close to a representable distribution (Fig. 11). The synthesised signal although non-representable (does not correspond to the distribution which produced it) has the desired features expressed by the mask. It is seen that it contains an impulse and a time variable reflection as it has been requested from the mask (Fig. 12).

## CONCLUSION

We have introduced filtering operations for signals and systems using concepts of TF analysis. This methodology uses masking or convolution in the TF domain and is based on non-parametric modeling using direct convolution or multiplication. Two types of TF filters have been discussed: *TF masking filters* and *TF convolution filters*.

Simulation of non-stationary systems and synthesis of non-stationary signals from filtered distributions have been developed using TF filters. This type of processing requires the evaluation of the decompositions of the input signal, the system response and the inverse transform of the output signal and hence requires considerable computation when compared to one dimensional filtering methods. However, provides a tool for detailed filtering operations, which can not realised with one dimensional methods. Care must be taken because the filtered distribution may not be representable by a synthesised signal. This is because TF filtering may disrupt essential parts of the distribution necessary for meaningful reconstruction.

Recursive linear TF analysis has been used because it is simple to implement and provides means of synthesis and simulation of non-stationary systems. A method based on Fourier analysis has been developed to produce non-stationary signals with desirable time and frequency characteristics. This method is faster when compared to non-recursive realisations and yields a simple synthesis procedure. A considerable reduction in computation of TF filtering operation has been achieved.

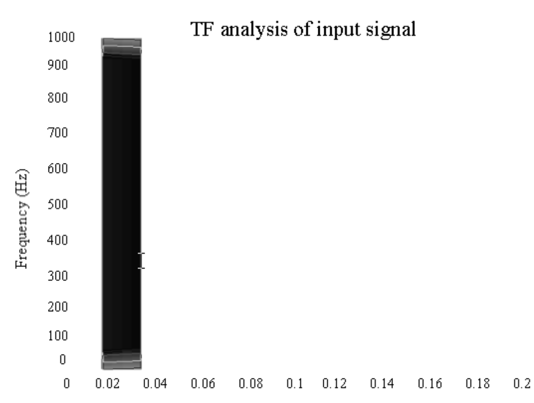


Figure 9. RDFS decomposition of the input signal.

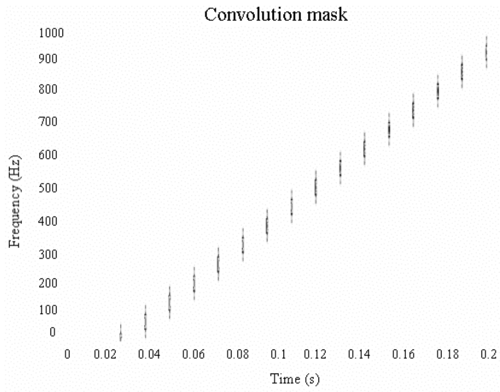


Figure 10. RDFS decomposition of the designed mask.

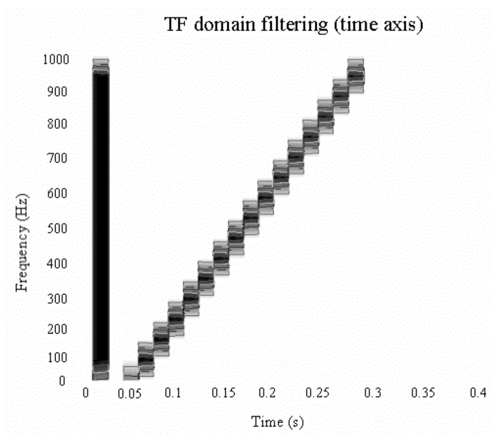


Figure 11. The result of filtering in the TF domain using convolution along the time axis.

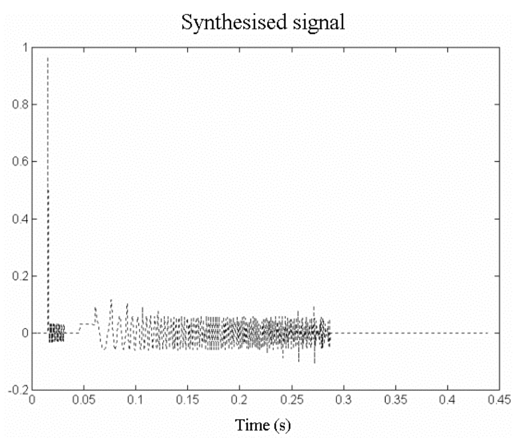


Figure 12. The synthesised signal.

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